SOLUTIONS

Joint Entrance Exam | IITJEE-2019

8th April 2019 | Evening Session

Joint Entrance Exam | JEE Mains 2019

PART-A	PHYSICS

1.(4) $B_{px} = \frac{\mu_0}{4\pi}\frac{2M}{\left(d/2\right)^3}\hat{i}$

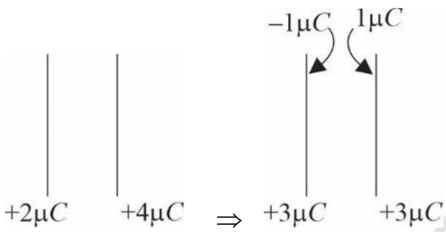
$B_{py} = \frac{\mu_0}{4\pi}\frac{(2M)}{\left(d/2\right)^3}\hat{j}$

Net magnetic field, $\vec{B} = \frac{\mu_0}{4\pi}\frac{2M}{\left(d/2\right)^3}\left(\hat{i}+\hat{j}\right)$

since $\vec{v} \parallel \vec{B}$

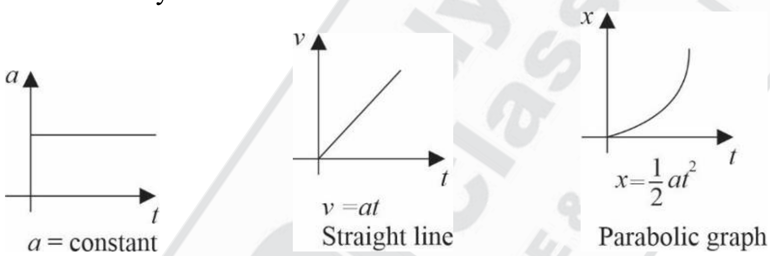
Hence force = 0

2.(4)



Pot. Diff $= \frac{Q}{C} = \frac{1\mu C}{1\mu F} = 1V$

3.(4) For uniformly accelerated motion



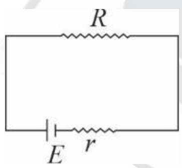
4.(4) $a \rightarrow$ isobaric

$b \rightarrow$ isothermal

$c \rightarrow$ adiabatic

$d \rightarrow$ isochoric

5.(4)



$i = \frac{E}{R+r}$

$P = i^2 R = \frac{E^2 R}{(R+r)^2}$

For maximum power

$\frac{dP}{dR} = 0 \Rightarrow R = r$

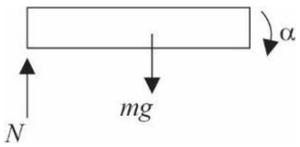
6.(2) Conserving momentum

$m_1\vec{v}_1 + m_2\vec{v}_2 = m_1\vec{v}_3 + m_2\vec{v}_4$

$m_1 v_1 + \frac{m_1 v_2}{2} = m_1 \frac{v_1}{2} + \frac{m_1}{2} v_4$

$\Rightarrow \quad \frac{v_1}{2} = \frac{v_4 - v_2}{2} \Rightarrow v_1 = v_4 - v_2$

7.(4)



The block will experience angular acceleration for a very small duration of time

$mg\frac{L}{2} = \frac{mL^2}{3}\alpha \Rightarrow \alpha = \frac{3g}{2L}$

$\omega = \alpha \Delta T \Rightarrow \omega = \frac{3g}{2L}T$

Time taken to reach ground $= \frac{2h}{g} = 1\text{sec}.$

angle rotated, $\theta = \frac{3g}{2L}T$

$\Rightarrow \quad \theta = \frac{3\times 10}{2\times 0.3}\times 0.01 = \frac{1}{2} = 0.5 \text{ rad}$

8.(1) Limit of resolution $= 1.22\frac{\lambda}{D}$

$= \frac{1.22\times 500\times 10^{-9}}{200\times 10^{-2}} = 305\times 10^{-9}$ rad.

9.(3) $KE_{sph} = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{2}{5}mR^2\right)\left(\frac{v^2}{R^2}\right)$

$= \frac{1}{2}mv^2\left(1+\frac{2}{5}\right) = \frac{7}{5}\left(\frac{1}{2}mv^2\right)$

Maximum ht = h_{sph}

$mgh_{sph} = \frac{7}{5}\left(\frac{1}{2}mv^2\right) \Rightarrow h_{sph} = \frac{7}{5}\left(\frac{v^2}{2g}\right)$

Similarly

$h_{cyl} = \frac{3}{2}\left(\frac{v^2}{2g}\right)$

$\Rightarrow \quad \frac{h_{sph}}{h_{cyl}} = \frac{7/5}{3/2} = \frac{14}{15}$

10.(2) $\vec{B} = 1.6\times 10^{-6}\cos\left(2\times 10^7 z + 6\times 10^{15} t\right)\left(2\hat{i} + \hat{j}\right)\frac{Wb}{m^2}$

$E_0 = cB_0$

$= 3\times 10^8 \times 1.6\times 10^{-6} = 4.8\times 10^2$

From the equation of $\vec{B}$ we can conclude that the direction of wave is $-\hat{k}$

Direction of $\vec{E} =$ direction of $\vec{B} \times \vec{v}$

$\left(2\hat{i} + \hat{j}\right) \times \left(-\hat{k}\right) = -\hat{i} + 2\hat{j}$

Hence

$\vec{E} = 4.8 \times 10^2 \cos\left(2 \times 10^7 z + 6 \times 10^{15} t\right)\left(-\hat{i} + 2\hat{j}\right) V/m$

11.(3) $[S] = \left[MT^{-2}\right]$

$[I] = \left[ML^2\right]$

$[h] = \left[ML^2T^{-1}\right]$

Linear momentum, $[P] = \left[MLT^{-1}\right]$

$[P] = [S]^a [I]^b [h]^c$

$= \left[M^{a+b+c} L^{2b+2c} T^{-2b-c}\right]$

Comparing we get

$a + b + c = 1$

$2b + 2c = 1$

$-2b - c = -1$

Solving we get

$a = 1/2, b = 1/2, c = 0$

12.(3) $V_E = 64 V_m \Rightarrow R_E = 4R_M$

$g_E = G.\rho \frac{4\pi}{3} R_E \Rightarrow \frac{g_M}{g_E} = \frac{R_M}{R_E} \Rightarrow g_M = \frac{g_E}{4}$

$E \propto v_e^2 \Rightarrow E \propto gR$

$\frac{E_M}{E_E} = \frac{g_M}{g_E} \frac{R_M}{R_E} \Rightarrow \frac{E_M}{E} = \frac{1}{4} \times \frac{1}{4}$

$\Rightarrow \quad E_M = \frac{E}{16}$

13.(1) $\left|\vec{A}_1 + \vec{A}_2\right|^2 = \left|\vec{A}_1\right|^2 + \left|\vec{A}_2\right|^2 + 2\left|\vec{A}_1\right|\left|\vec{A}_2\right| \cos\theta$

$25 = 9 + 25 + 2 \times 15 \cos\theta$

$\Rightarrow \cos\theta = \frac{-3}{10}$

$\Rightarrow \vec{A}_1.\vec{A}_2 = \left|\vec{A}_1\right|\left|\vec{A}_2\right| \cos\theta = 3 \times 5 \times \left(\frac{-3}{10}\right) = \frac{-9}{2}$

$\left(2\vec{A}_1 + 3\vec{A}_2\right).\left(3\vec{A}_1 - 2\vec{A}_2\right) = 6\left|\vec{A}_1\right|^2 - 3\left|\vec{A}_2\right|^2 + 5\vec{A}_1.\vec{A}_2$

$= 54 - 150 + 5\left(\frac{-9}{2}\right) = -118.5$

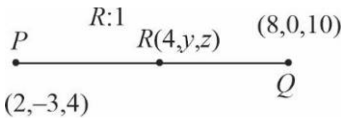
14.(1) Radius

$R \propto A^{1/3}$

$\Rightarrow$ Volume, $V \propto A$

mass $\propto A$

hence mass densities i.e. $\frac{M}{V}$ will be equal

15.(3) 

$\Delta l = \frac{Fl}{AY}$

$\frac{L_1}{A_1Y_1} = \frac{L_2}{A_2Y_2} \Rightarrow \frac{2}{R^2 \times 7} = \frac{1.5}{4 \times 4}$

$\Rightarrow \quad R = \sqrt{\frac{64}{21}} \simeq 1.7mm$

16.(1) Since current leads emf, hence circuit will have capacitor and resistor.

$X_c = R \Rightarrow \frac{1}{100C} = 10^3 \Rightarrow C = 10\mu F$

17.(1) $V_{rms} = \sqrt{\frac{3k_bT}{m}} = \sqrt{\frac{3k_bN_AT}{M}}$

$V_e = \sqrt{2gR}$

Equating the two we get

$T = 10^4 K$

18.(2) $x = \frac{m\left(\frac{a}{4}\right) + m\left(\frac{a}{4}\right) + m\left(\frac{3a}{4}\right)}{3m} = \frac{5b}{12}$

$y = \frac{m\left(\frac{b}{4}\right) + m\left(\frac{b}{4}\right) + m\left(\frac{3b}{4}\right)}{3m} = \frac{5b}{12}$

19.(1) $i = \frac{V}{R_{eq}} = \frac{3}{6} = 0.5A$

$\Delta V_{AJ} = iR_{AJ} = (0.5)(0.5) = 0.25V$

20.(2) $T = 2\pi\sqrt{\frac{l}{g}} \quad \Rightarrow g = \frac{4\pi^2 l}{T^2}$

$\frac{\Delta g}{g} \times 100 = \frac{\Delta l}{l} \times 100 + 2\frac{\Delta T}{T} \times 100$

$= \frac{1}{550} \times 100 + 2 \times \frac{1}{30} \times 100$

$\approx 6.8\%$

21.(4) Amplitude $A = A_0 e^{-bt/2m}$

$$\frac{A_0}{2} = A_0 e^{-\frac{b}{2m}(2)}$$

$$\frac{A_0}{1000} = A_0 e^{\frac{-b}{2m}t}$$

Solving we get , $t = 20s$.

22.(1) Using lens formula

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} - \frac{1}{-30} = \frac{1}{20} \Rightarrow \frac{1}{v} = \frac{1}{60}$$

$v = 60cm$

Since the image is formed at the same point, it must be at center of curvature of mirror.

$R_{mirror} = 20cm \Rightarrow f_{mirror} = 10cm$

$\Rightarrow$ Max. distance for virtual image = 10cm

23.(4) $R_{eq} = \frac{160}{3}$

$$i = \frac{V}{R_{eq}} = \frac{15}{160/3} = \frac{9}{32}$$

24.(4) $E \propto \frac{1}{r}$

$F \propto \frac{1}{r}$

$a \propto \frac{1}{r}$

$vdv \propto \frac{1}{r}dr$

$$v^2 \propto \ln\left(\frac{r}{r_0}\right)$$

$$v \propto \sqrt{\ln\left(\frac{r}{r_0}\right)}$$

25.(4) $E = Ax + B$

$dV = -E\ dx$

$$\int_{V_1}^{V_2} dV = -\int_{1}^{-5} (20x + 10)dx$$

$V_2 - V_1 = -180$

$V_1 - V_2 = 180V$

26.(4) $d = \sqrt{2Rh_T} + \sqrt{2Rh_R}$

$$50 \times 10^3 = \sqrt{2 \times 6.4 \times 10^6 \times 70} + \sqrt{2 \times 6.4 \times 10^6 \times h_R}$$

$\Rightarrow$ $h_R = 32m$

27.(1) $\Delta I_C = \frac{V_{CC}}{R_C}$

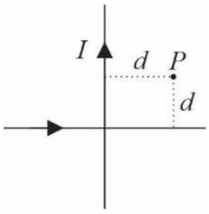
$\beta = \frac{\Delta I_C}{\Delta I_B}$

$\Delta I_B = \frac{\Delta I_C}{\beta} = \frac{V_{CC}}{R_C.\beta}$

Putting the values we get

$\Delta I_B = 40\mu A$

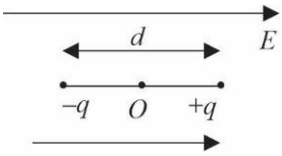
28.(2)



$\vec{B} = \frac{\mu_0}{2\pi}\frac{I}{d}\left(\hat{k}\right) + \frac{\mu_0}{2\pi}\frac{I}{d}\left(-\hat{k}\right)$

$\vec{B} = 0$

29.(3)



$\tau = PE\sin\theta$

$I\alpha = q.dE\sin\theta$

$\alpha = \frac{qdE}{I}\sin\theta$

As θ is small $\alpha = \frac{qdE}{I}\theta$

$\omega = \sqrt{\frac{qdE}{I}}$, where, $I = 2.m\frac{d^2}{4}$

30.(1) $m_A.u_A = m_B.u_B - m_C u_C$

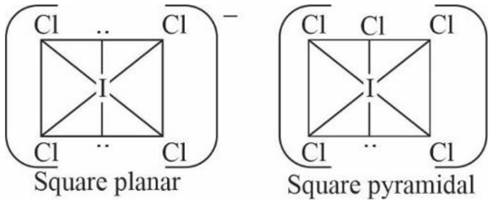
As $m_B = m_C$, $\frac{h}{\lambda_A} = m_B.u_B - m_B\frac{u_B}{2}$

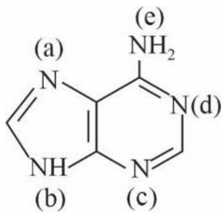
$\frac{h}{\lambda_A} = \frac{1}{2}m_B.u_B = \frac{1}{2}\frac{h}{\lambda_B}$

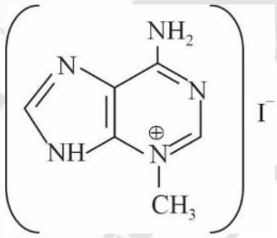
$\lambda_B = \frac{\lambda_A}{2}$

$\lambda_C = \lambda_A$

PART-B	CHEMISTRY

1.(2) 

2.(2) 

Ione pair on "N" marked as "C" is most nucleophilic and form  I^- with CH_3I

3.(4) CH_4

$n_c = 1$ mole

$n_H = 4$mole

$$\%C = \frac{n_C}{n_C + n_H} \times 100 = \frac{1}{1+4} \times 100 = 20\%$$

4.(1) $C_2^{-2} \longrightarrow \sigma 1s^2, \sigma^* 1s^2, \sigma 2s^2, \sigma^* 2s^2, \left[\pi 2Px^2 = \pi 2Py^2\right] \sigma 2Pz^2$

Bond order $= \frac{10-4}{2} = 3(\text{diamagnetic})$

$N_2^{-2} \rightarrow \sigma 1s^2, \sigma^* 1s^2, \sigma 2s^2, \sigma^* 2s^2, \left[\pi 2P_x^2 = \pi 2P_y^2\right] \sigma 2P_z^2 \left[\pi^* 2P_x^1 = \pi^* 2P_y^1\right]$

$B.O = \frac{10-6}{2} = 2(\text{Paramagnetic})$

$O_2^{-2} \rightarrow \sigma 1s^2, \sigma^* 1s^2, \sigma 2s^2, \sigma^* 2s^2, \sigma 2P_z^2 \left[\pi 2P_x^2 = \pi 2P_y^2\right]\left[\pi^* 2P_x^2 = \pi^* 2P_y^2\right]$

$B.O = \frac{10-8}{2} = 1(\text{Diamagnetic})$

$O_2 \rightarrow \sigma 1s^2, \sigma^* 1s^2, \sigma 2s^2, \sigma^* 2s^2, \sigma 2P_z^2 \left[\pi 2P_x^2 = \pi 2P_y^2\right]\left[\pi^* 2P_x^1 = \pi^* 2P_y^1\right]$

$B.O = \frac{10-6}{2} = 2(\text{Paramagnetic})$

$B.O \propto \frac{1}{BL}$

5.(1) $\Delta U = nC_v\Delta T = \frac{5 \times 28 \times 100}{1000} = 14KJ$

$\Delta(PV) = nR\Delta T = \frac{5 \times 8 \times 100}{1000} = 4KJ$

6.(3) Interstitial compound are almost inert.

7.(3)
$$\underset{\text{Impure}}{Ni(s)} + 4CO(g) \xrightarrow{50°C} [Ni(CO)_4]$$
$$[Ni(CO)_4] \xrightarrow{200°-250°C} \underset{\text{pure}}{Ni(s)} + 4CO(g)$$

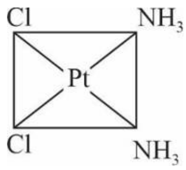
8.(4)
$$\frac{d[B]}{dt} = K_1[A] - K_2[B]$$
$$\frac{d[B]}{dt} = 0$$
$$\Rightarrow \quad [B] = \frac{K_1}{K_2}[A]$$

9.(1) CF_3 is very strong-I group hence it will favour anti-Markownikaff's product.

$$CF_3 - CH = CH_2 \xrightarrow{HX} CF_3 - CH_2 - CH_2^+ \longrightarrow CF_3 - CH_2 - CH_2X$$ (anti-Markownikoff's product)

10.(4) According to Fajan's rule when size of cation decrease, then covalent character increase.
So BeX_2 is covalent in nature.

11.(2)



12.(3)
$$K_{eq} = \frac{[SO_3]^2}{[SO_2]^2[O_2]} \qquad ...(1)$$
$$K_1 = \frac{[SO_2]}{[O_2]} = 10^{52}$$
$$K_2 = \frac{[SO_3]^2}{[O_2]^3} = 10^{129} \qquad ...(2)$$
$$K_1^2 = \frac{[SO_2]^2}{[O_2]^2} = 10^{104} \qquad ...(3)$$
$$K_{eq} = \frac{K_2}{(K_1)^2} = \frac{10^{129}}{10^{104}} = 10^{25}$$

13.(1) Seliwanoff's test: Seliwanoff's reagent is (0.5%) resorcinol in 3N HCl. It gives red solution with fructose and sucrose but no change in colour with glucose.

14.(3) $119 \rightarrow$ Ununenium (Uue)

15.(3) Mass of fatty acid = 0.027g
Density = 0.9 g/cc

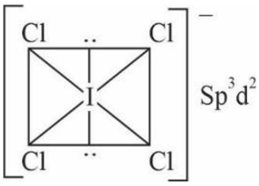
Volume of fatty acid $= \frac{0.027}{0.9} = 0.03 cm^3$

Area of plate $= \pi r^2 = 3 \times (10)^2 = 300 cm^2$

Volume of fatty acid = area of plate × height of fatty acid layer

$0.03\text{cm}^3 = 300 \times h \qquad \Rightarrow \; h = \dfrac{0.03}{300}\text{cm} = 10^{-4}\text{cm} = 10^{-6}\text{m}$

16.(1)

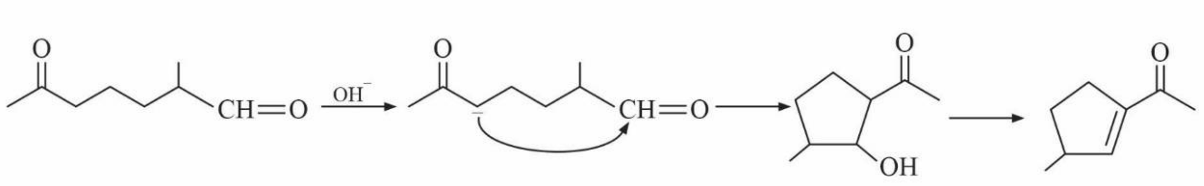


$ICl_2^- \rightarrow sp^3d$

$BrF_2^- \rightarrow sp^3d$

$IF_6^- \rightarrow sp^3d^3$

17.(3) It is example of intramolecular aldol condensation.



18.(3) $P.F. = \dfrac{\left(z_{eff} \times \frac{4}{3}\pi r_A^3\right)_A + \left(z_{eff} \times \frac{4}{3}\pi r_B^3\right)_B}{a^3}$

$2(r_A + r_B) = \sqrt{3}a$

$\Rightarrow \quad 2(r_A + 2r_A) = \sqrt{3}a$

$\Rightarrow \quad 2\sqrt{3}r_A = a$

$\Rightarrow \quad p.f. = \dfrac{1 \times \frac{4}{3}\pi r_A^3 + \frac{4}{3}\pi(8r_A^3)}{8 \times 3\sqrt{3}r_A^3} = \dfrac{9 \times \frac{4}{3}\pi}{8 \times 3\sqrt{3}} = \dfrac{\pi}{2\sqrt{3}}$

$P.F = \dfrac{\pi}{2\sqrt{3}} \times 100 = 90\%$

19.(1) Nylon-6 is derived from $NH_2-(CH_2)_5-\overset{\overset{O}{\|}}{C}-OH$

20.(2) In Friedel craft alkylation, the alkylated product obtained is more activated than reactant, hence undergoes poly substitution.

21.(1) $M = \dfrac{\text{Volume strength}}{11.2}$

$M = 1$

$\dfrac{W}{M.W.} \times \dfrac{1000}{V} = 1$

$\dfrac{W}{V} \times 100 = \dfrac{MW.}{10}$

$\% \dfrac{W}{V} = \dfrac{34}{10} = 3.4\%$

22.(1) $P_{gas} = K_H.X_{gas}$

$P_{gas} = K_H.\left(1 - X_{H_2O}\right)$

$P_{gas} = K_H - K_H.X_{H_2O}$

$y = c + mx$

For higher K_H, slow will be more negative

23.(2) $\left[Fe\left(CN\right)_6\right]^{-4}$

$Fe^{+2} \rightarrow 3d^6$

$CN^- \rightarrow$ S.F.L.

$t2g^{222}eg^{00}$

No. of unpaired $e^- = 0$

$B.M. = 0$

$\left[Fe\left(H_2O\right)_6\right]^{+2}$

$Fe^{+2} \rightarrow 3d^6$

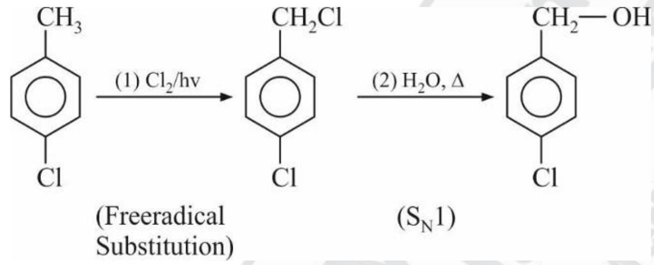
$H_2O \rightarrow$ W.F.L .

${t_{2g}}^{2,1,1}eg^{1,1}$

No. of unpaired $e^- = 4$

$\mu = \sqrt{4 \times 6} = \sqrt{24}$ B.M. $= 4.9$BM

24.(1)



25.(4) It is factual, taken from NCERT.

26.(1) $E = W + KE_{max}$

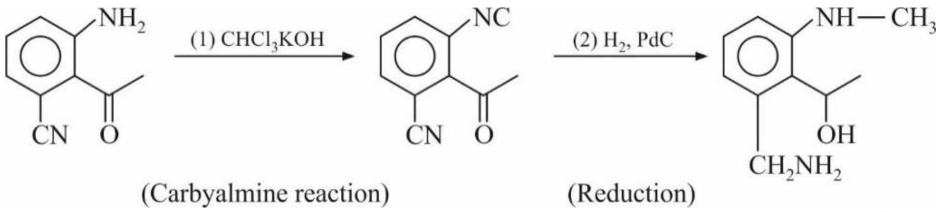
$\frac{hc}{\lambda} = W + \frac{P^2}{2m}$

$\left(W \rightarrow 0\right)$

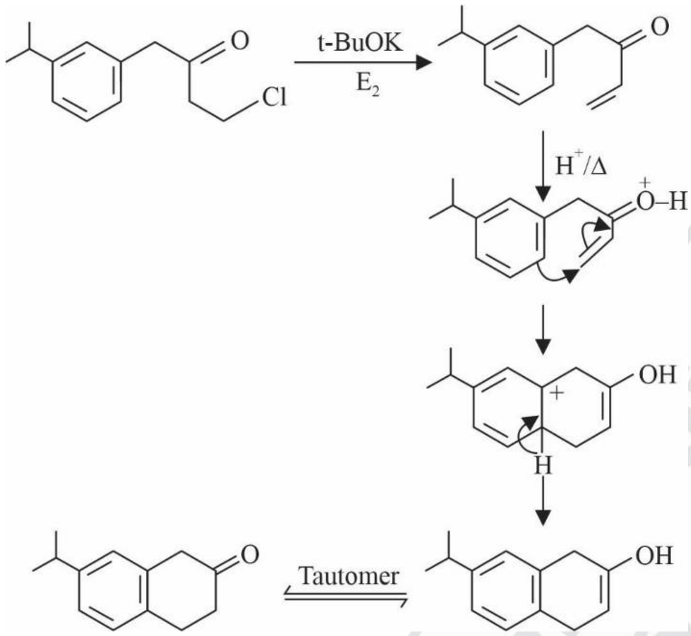
$\frac{\lambda_2}{\lambda_1} = \frac{P_1^2}{P_2^2}$

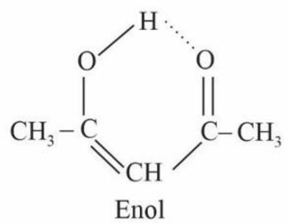
$\frac{\lambda_2}{\lambda} = \frac{P^2}{\left(1.5P\right)^2}$

$\frac{\lambda_2}{\lambda} = \frac{4}{9}$

27.(4) 

28.(4)



29.(1) 

Due to intramolecular H-bonding enol content is maximum

30.(3) $Fe^{2+} \rightarrow Fe \qquad \Delta G_1^0 = -2Fy$

$Fe^{3+} \rightarrow Fe \qquad \Delta G_2^0 = -3Fz$

$\therefore\ Fe^{2+} \rightarrow Fe^{3+}\ \Delta G_3^0 = -1FE^0$

$\Delta G_3^0 = \Delta G_1^0 - \Delta G_2^0$

$-FE^\circ = -2Fy + 3Fz \Rightarrow E^\circ = 2y - 3z$

For given cell reaction.

$Fe^{2+}(aq) + Ag^{+}(aq) \rightarrow Fe^{3+}(aq) + Ag(s)$

$E_{cell}^0 = x + 2y - 3z$

PART-C	MATHEMATICS

1.(3) $S=\frac{1}{2}+\frac{2}{2^2}+...........+\frac{20}{2^{20}}$

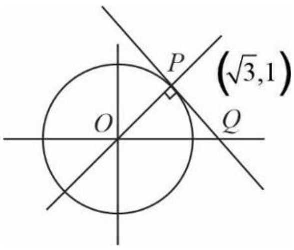
$\frac{S}{2}=\frac{1}{2^2}+\frac{2}{2^3}+...............+\frac{20}{2^{21}}$

$\frac{S}{2}=\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+..............+\frac{1}{2^{20}}-\frac{20}{2^{21}}$

$=\frac{1}{2}\frac{\left(1-\left(\frac{1}{2}\right)^{20}\right)}{1-\frac{1}{2}}-\frac{20}{2^{21}}$

$S=2-\frac{22}{2^{20}}=2-\frac{11}{2^{19}}$

2.(3)



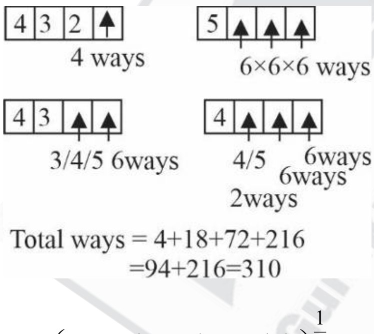
Slope of $OP=\frac{1}{\sqrt{3}}$, Slope of $PQ=-\sqrt{3}$

$y-1=-\sqrt{3}\left(x-\sqrt{3}\right)=-\sqrt{3}x+3$

$\Rightarrow \quad \sqrt{3}x+y=4$ and $Q\left(\frac{4}{\sqrt{3}},0\right)$

$\Delta OPQ=\frac{2}{\sqrt{3}}$

3.(2) Total ways $=4+8+72+216=94+216=310$



4.(4) $\lim_{x\to 0}\left(\frac{1+f(3+x)-f(3)}{1+f(2-x)-f(2)}\right)^{\frac{1}{x}}$

$\Rightarrow \; = e^{\lim_{x\to 0}\left(\frac{1+f(3+x)-f(3)}{1+f(2-x)-f(2)}-1\right)\frac{1}{x}}$

$=e^{f'(3)+f'(2)}=e^0=1$

5.(2) $L_1:(y-2)=-\frac{1}{2}(x-1)=x+2y-5=0$

$L_2:(y-3)=2(x-4)=2x-y-5=0$

Put h,k in both lines

$(h,k)=(3,1)\Rightarrow \frac{k}{h}=\frac{1}{3}$

6.(1) $\frac{dy}{dx}=\frac{2y}{x^2}\Rightarrow \ln y=-\frac{2}{x}+\ln C$

Passes through $(1,1)$

$0=\frac{-2}{1}+\ln C, \ln C=2$

$\ln|y|=-\frac{2}{x}+2$

$x\ln|y|=2(x-1)$

7.(4) $x^2+4x-5=0$

$(x+5)(x-1)=0$

Required point in quadrant first is (1, 2)

Required equation is $x-y+1=0$ and now check option]

8.(4) $D=4(1+3m)^2-4(1+m^2)(1+8m)$

$=4\left(1+9m^2+6m-\left(1+8m+m^2+8m^3\right)\right)$

$=4\left(8m^2-2m-8m^3\right)$

$=-8m(2m-1)^2<0$

Hence infinitely many values

9.(No answer)

${}^6C_3\left(\frac{1}{x^{1+\log_{10}x}}\right)^{3/2}x^{1/4}=200$

$x^{\frac{1}{4}-\frac{3}{2}(1+\log_{10}x)}=10$

$\left(\frac{1}{4}-\frac{3}{2}(1+t)\right)t=1$

Where $t=\log_{10}x$

$6t^2+5t+4=0$

$D<0$, No solution.

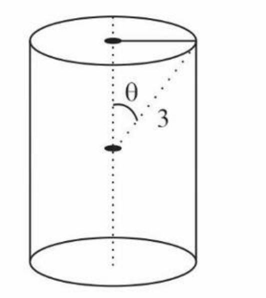
10.(3) $x-2y+kz=1$ …(i)

$2x+y+z=2$ …(ii)

$3x-y-kz=3$ …(iii)

Add (i) & (iii)

$4x-3y-4=0$

11.(4) 

$h = 2(3\cos\theta)$

$r = 3\sin\theta$

$v = \pi r^2 h$

$= \pi 9\sin^2\theta.6\cos\theta$

$V = 54\pi\sin 2\theta\cos\theta$

$\frac{dv}{d\theta} = 0$

$\Rightarrow \quad 2\sin\theta\cos^2\theta - \sin^3\theta = 0$

$\Rightarrow \quad 2s\left(1-s^2\right) - s^3 = 0$

$\Rightarrow \quad 2s - 2s^3 - s^3 = 0$

$\Rightarrow \quad 2s - 3s^3 = 0$

$\Rightarrow \quad s = 0$ or $2 - 3s^2 = 0$

$s = \pm\sqrt{\frac{2}{3}}$

$\therefore \quad \cos\theta = \sqrt{1-\frac{2}{3}} = \frac{1}{\sqrt{3}}$

$h = 6\left(\frac{1}{\sqrt{3}}\right)$

$h = 2\sqrt{3}$

12.(4) $f_1(x) = \frac{a^x + a^{-x}}{2}$

$\Rightarrow \quad f_1(x+y) + f_1(x-y)$

$= 2f_1(x)f_1(x)$

13.(2) Let hyperbola be

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

$\Rightarrow \quad \frac{16}{a^2} - \frac{36}{b^2} = 1$

also $\frac{16}{a^2} - \frac{36}{b^2} = 1$ (from $e = 2$)

$\Rightarrow \quad a^2 = 4$ and $b^2 = 12$

Equation of tangent at $(4,6)$ is

$2x - y - 2 = 0.$

14.(3) a, b, c are in $A.P$

$\angle C = 2\angle A$

$\Rightarrow \quad \sin C = \sin 2A$

$\frac{\sin C}{\sin A} = 2\cos A$

$\Rightarrow \quad \frac{c}{a} = 2\frac{\left(b^2 + c^2 - a^2\right)}{2bc}$

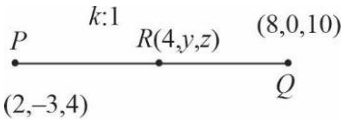
Put $a = b - d, \quad c = b + d,\ d > 0$

$\Rightarrow \quad d = \frac{b}{5}$

$a = \frac{4}{5}b, \qquad C = \frac{6b}{5}$

$a : b : c = 4 : 5 : 6$

15.(3)



$\frac{8k+2}{k+1} = 4 \qquad \Rightarrow\ k = \frac{1}{2}$

$\Rightarrow \quad y = -2 \ \ z = 6$

$\therefore \quad R(4, -2, 6)$ Distance from origin $= 2\sqrt{14}$.

16.(2) $\int \frac{dx}{x^3\left(1+x^6\right)^{2/3}} \Rightarrow \int \frac{dx}{x^7\left(1+\frac{1}{x^6}\right)^{2/3}} \Rightarrow \left(1+\frac{1}{x^6}\right) = t$

$-\frac{6}{x^7}dx = dt \Rightarrow -\frac{1}{6}\int \frac{dt}{t^{2/3}}$

$-\frac{1}{6}\left(\frac{t^{1/3}}{\frac{1}{3}}\right) = -\frac{1}{2}\left[\left(1+\frac{1}{x^6}\right)^{1/3}\right] + C = -\frac{1}{2}\frac{\left(1+x^6\right)^{1/3}}{x^6} + C = xf(x)\left(1+x^6\right)^{1/3}$

$f(x) = -\frac{1}{2x^3}$

17.(4) $b^2 = ac$

roots of $ax^2 + 2bx + c = 0$ are equal i.e. $-\frac{b}{a}$

$d\left(-\frac{b}{a}\right)^2 + 2e\left(-\frac{b}{a}\right) + f = 0$

$db^2 - 2bea + fa^2 = 0$

$dc - 2eb + fa = 0$

Divide by ac

$\frac{dc}{ac} - \frac{2eb}{b^2} + \frac{fa}{ac} = 0 \Rightarrow \frac{d}{a} - \frac{2eb}{b^2} + \frac{fa}{ac} = 0 \Rightarrow \frac{d}{a} - \frac{2e}{b} + \frac{f}{c} = 0$

$\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in A.P.

18.(1)

p	q	$p \vee q$	$\sim q$	$p \vee \sim q$	$(p \vee q) \rightarrow (p \vee (\sim q))$
T	T	T	F	T	T
T	F	T	T	T	T
F	T	T	F	F	F
F	F	F	T	T	T

Hence not a tautology

19.(1) $f(x) = \int_0^x g(t)dt$

$g(x)$ is even $fn. \therefore$ $f(x)$ is odd $fn.$

Also $f'(x) = g(x)$

$f(x+5) = g(x)$

$f(5-x) = g(-x) = g(x) = f(x+5)$

Now $\int_0^x f(t)dt$

$t = 5 + z$ $\qquad\qquad dt = dz$

$\int_{-5}^{x-5} f(5+z)dz = \int_{-5}^{x-5} g(z)dz$

$= \int_{-5}^{x-5} f'(z)dz = f(x-5) - f(-5)$

$= f(5) - f(5-x)$

$= f(5) - f(5+x)$

$= \int_{x+5}^{5} f'(t)dt = \int_{x+5}^{5} g(t)dt$

20.(3) $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 2 & x \\ 1 & -1 & 1 \end{vmatrix}$

$|\vec{a} \times \vec{b}| = \sqrt{2x^2 - 2x + 38} = r$

$\Rightarrow \qquad r \geq \sqrt{\frac{75}{2}}$

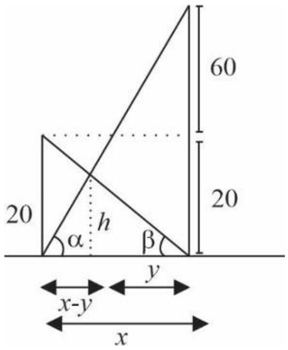
$\Rightarrow \qquad r \geq 5\sqrt{\frac{3}{2}}$

21.(4) $y = f(f(f(x))) + (f(x))^2$

$\frac{dy}{dx} = f'(f(f(x))).f'(f(x)).f'(x) + 2f(x)f'(x)$

Put $x = 1 \Rightarrow f'(f(f(1))).f'(f(1)).f'(1) + 2f(1).f'(1) = 27 + 6 = 33$

22.(3) Height of two tower are 20 m 80 m ………………



$\frac{h}{y} = \tan\beta \quad \Rightarrow \quad \frac{h}{y} = \frac{20}{x}, \frac{h}{x-y} = \frac{80}{x} \quad \Rightarrow \quad \frac{hx}{20} = y. \frac{hx}{80} = x - y$

$\frac{hx}{20} + \frac{hx}{80} = x \quad \Rightarrow \quad \frac{h}{20} + \frac{h}{80} = 1$

$5h = 80 \quad \Rightarrow \quad h = 16$

23.(2) $1 - \frac{1}{2^n} > \frac{9}{10} \Rightarrow \frac{1}{2^n} \Rightarrow 2^n > 10$ $\quad \therefore$ minimum value of n is 4

24.(3) $z = \frac{\sqrt{3}}{2} + \frac{i}{2} = \cos\frac{\pi}{6} + i\sin\frac{\pi}{6}$

$\Rightarrow \quad z^5 = \frac{-\sqrt{3}}{2} + \frac{i}{2}$

$z^8 = -\frac{1}{2} - \frac{i\sqrt{3}}{2}$

$\Rightarrow \left(1 + iz + z^5 + iz^8\right)^9$

$= \left(\frac{1}{2} + \frac{i\sqrt{3}}{2}\right)^9 = \cos 3\pi + i\sin 3\pi$

$= -1.$

25.(3) $\frac{41 + 45 + 54 + 57 + 43 + x}{6} = 48 \Rightarrow x = 48$

$\sigma^2 + 48^2 = \frac{1}{6}\left(41^2 + 45^2 + 54^2 + 57^2 + 43^2 + 48^2\right)$

$\sigma^2 = \frac{14024}{6} - 2304 = \frac{7012}{3} - 2304 = \frac{7012 - 6912}{3} = \frac{100}{3}$

$\sigma = \frac{10}{\sqrt{3}}$

26.(3) $be = 5\sqrt{3} \Rightarrow b^2e^2 = 75$

$b^2 - a^2 = 75$

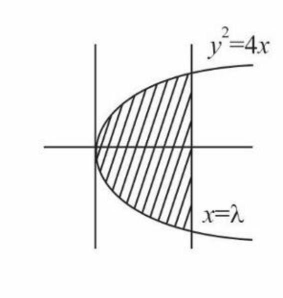
$(b - a)(b + a) = 75$

$b + a = 15$

$b = 10, a = 5$

$LR = \frac{2a^2}{b} = \frac{2 \times 25}{10} = 5$

27.(1)



$y^2 = 4x$

$S(I) = 2\int_0^{\lambda} 2\sqrt{x}dx = \left.\frac{4x^{3/2}}{3/2}\right]_0^{\lambda} = \frac{8}{3}\lambda^{3/2}$

$\frac{S(\lambda)}{S(4)} = \frac{2}{5} \Rightarrow \frac{\lambda^{3/2}}{4^{3/2}} = \frac{2}{5}$

$\lambda = 4\left(\frac{2}{5}\right)^{2/3} = 4\left(\frac{4}{25}\right)^{1/3}$

28.(3) $\begin{vmatrix} 1 & 1 & 1 \\ 2 & b & c \\ 4 & b^2 & c^2 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ 2 & (b-2) & (c-2) \\ 4 & (b^2-4) & (c^2-4) \end{vmatrix} = \begin{vmatrix} (b-2) & (c-2) \\ (b^2-4) & (c^2-4) \end{vmatrix}$

$= (b-2)(c-2)\begin{vmatrix} 1 & 1 \\ b+2 & (c+2) \end{vmatrix}$

$|A| = (b-2)(c-2)(c-b)$

$2, b, c$ are in A.P.

$2, 2+d, 2+2d$

$|A| = d(2d)d = 2d^3 \in [2,16] \Rightarrow d^3 \in [1,8] \Rightarrow d \in [1,2] \Rightarrow 2d \in [2,4]$

$2+2d \in [4,6]$

29.(4) $(x+y+z-1)+\lambda(2x+3y+4z-5)=0$

$(1+2\lambda)x+(1+3\lambda)y+(1+4\lambda)z-1-5\lambda=0$

$\Rightarrow \quad \lambda = -\frac{1}{3}$

Plane is $x-z+2=0$

$\vec{r}.(\hat{i}-\hat{k})+2=0$

30.(1) $f(x)\begin{cases} -(x+1), & -1 \le x < 0 \\ x & , 0 \le x < 1 \\ 2x & , 1 \le x < 2 \\ x+2 & , 2 \le x < 3 \\ x+3 & , x = 3 \end{cases}$

$\therefore$ Discontinuous at

$x = 0, 1, 3$